High-Resolution Frequency Control and Thermometry for Precise Measurements of Helium Density+

+ Research sponsored by NASA's Microgravity Research Division. JPL is supported by a contract from NASA.

Principal Investigator: Don Strayer JPL

Co-Investigators: Nai-Chang Yeh, Caltech
Talso Chui, JPL
Mark Lysek, JPL

Research Associate: Wen Jiang, NRC, JPL

Technical Engineers: Nils Asplund, Caltech
John Gatewood, Caltech

Student: Jeffrey Huynh

Approach

- **High-Q microwave resonator filled with helium**

$$\frac{\Delta f}{f_0} = \frac{f - f_0}{f_0} = -\frac{\int_{V_0} (\varepsilon - \varepsilon_0) |E|^2 \, dV}{\int_{V_0} \varepsilon_0 |E|^2 \, dV}$$

- **Clausius-Mossotti relation** $\dfrac{\varepsilon - 1}{\varepsilon + 2} = \dfrac{4\pi\alpha_0\rho}{3M}$

μg: $\varepsilon = \varepsilon(T)$ $\qquad \dfrac{f(T) - f_0}{f_0} = \dfrac{\varepsilon(T) - \varepsilon_0}{\varepsilon_0} \overset{\text{CM}}{\Rightarrow} \rho(T)$

1g: $\varepsilon = \varepsilon(T, z)$ $\qquad \dfrac{f(T) - f_0}{f_0} \overset{\textbf{Deconvolution}}{\Rightarrow} \varepsilon(T) \overset{\text{CM}}{\Rightarrow} \rho(T)$

Research Goal

- To investigate the superfluid to normal fluid transition near the lambda point T_λ in ^4He with precision measurements of helium density;

- To verify the universality of phase transitions near $T_\lambda(P)$ for different pressures .

Other Applications of the Precision Measurement Techniques

- Determining the precise phase boundaries $T_{\lambda}(X)$ and $T_{\sigma}(X)$ for the phase diagram of the ^3He-^4He mixture by precision measurements of X and T;

- Investigating the critical dynamics near the tricritical point T_t in the ^3He-^4He mixture;

- Investigating the superfluid and normal fluid interface under gravity.

Principle

- **High-Q microwave resonator filled with helium**

$$\frac{\Delta f}{f_0} = \frac{f - f_0}{f_0} = -\frac{\int_{V_0} (\varepsilon - \varepsilon_0) |E|^2 \, dV}{\int_{V_0} \varepsilon_0 |E|^2 \, dV}$$

f: **resonant frequency;**
ε: **dielectric constant;**
V_0: **volume of the cavity;**
E: **electric field of the resonant mode.**

- **Clausius-Mossotti relation**

$$\frac{\varepsilon-1}{\varepsilon+2}=\frac{4\pi\alpha_0\rho}{3M}$$

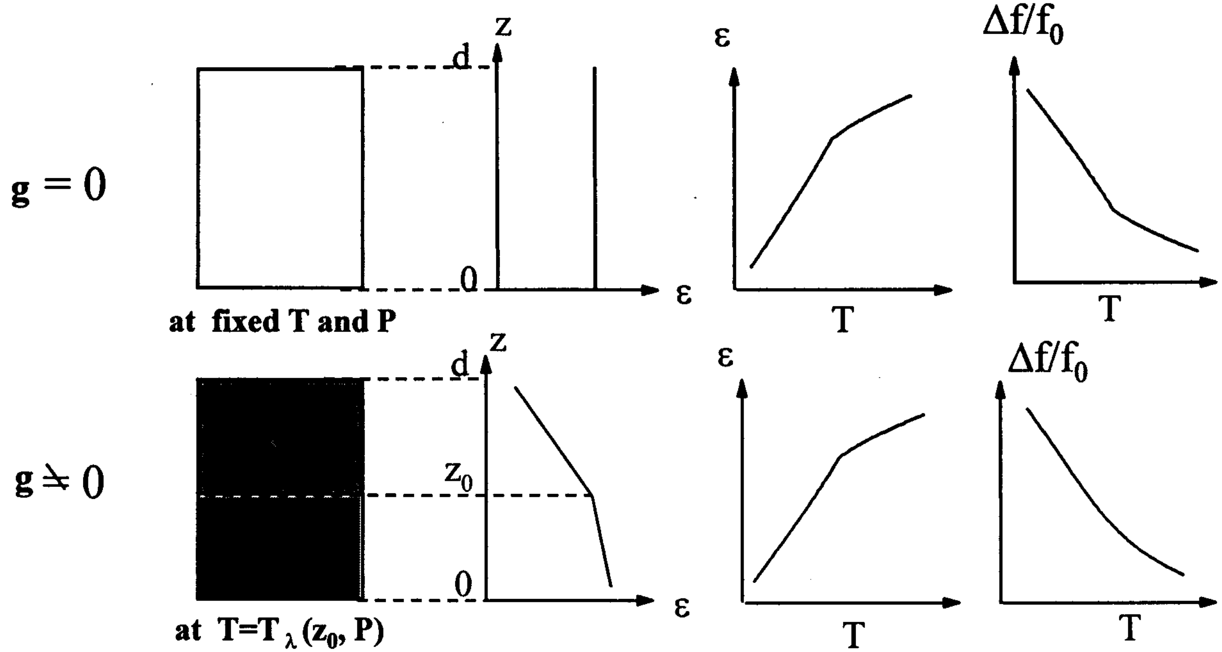
ε: **dielectric constant;**
ρ: **density;**
α_0: **polarizability;**
M: **molecular weight.**

Gravity Effect

$$T_\lambda = T_\lambda(0) + \gamma_0 z$$

$$\gamma_0 = 1.273 \times 10^{-6} K / cm$$

- **Gravity induces a ρ (and ε) profile in the cavity;**
- **For $T_\lambda(0)<T<T_\lambda(d)$, superfluid/normal fluid interface appears at $z_0=[T-T_\lambda(0)]/\gamma_0$.**



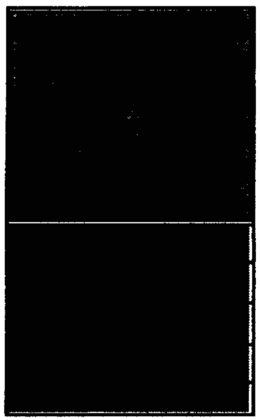
Deconvolution Algorithm for Resolving Gravity-Induced Density Profile

$$\frac{\Delta f}{f_0}(T) = \frac{f(T)-f_0}{f_0} = -\frac{\int [\varepsilon(z,T)-\varepsilon(z,T_\lambda(0))]\sin^2(\frac{l\pi z}{d})\,dz}{\int \varepsilon(z,T_\lambda(0))\sin^2(\frac{l\pi z}{d})\,dz}$$

Discretization:

Define $f_0 = f(T=T_\lambda(0)),\ \varepsilon_0 = \varepsilon(T_\lambda(z)),\ t_i^j = t(T_j, z_i) \equiv T_j - T_\lambda(z_i),\ z_i = i\frac{d}{N},\ i,j = 1,\ldots,N,$

$$A\,\mathbf{x} = \mathbf{b} \quad \Longrightarrow \quad \mathbf{x} = (A^T A)^{-1} A^T \mathbf{b}$$



$$\mathbf{x} = \varepsilon - \varepsilon_0, \quad \mathbf{b} = (\varepsilon_0 N/2)\mathbf{C}$$

$$\varepsilon = \begin{pmatrix} \varepsilon(-dt) \\ \varepsilon(-\frac{N-1}{N}dt) \\ . \\ . \\ \varepsilon(dt) \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} \frac{\Delta f}{f_0}(T_1) \\ \frac{\Delta f}{f_0}(T_2) \\ . \\ . \\ \frac{\Delta f}{f_0}(T_N) \end{pmatrix}$$

Numerical Simulations